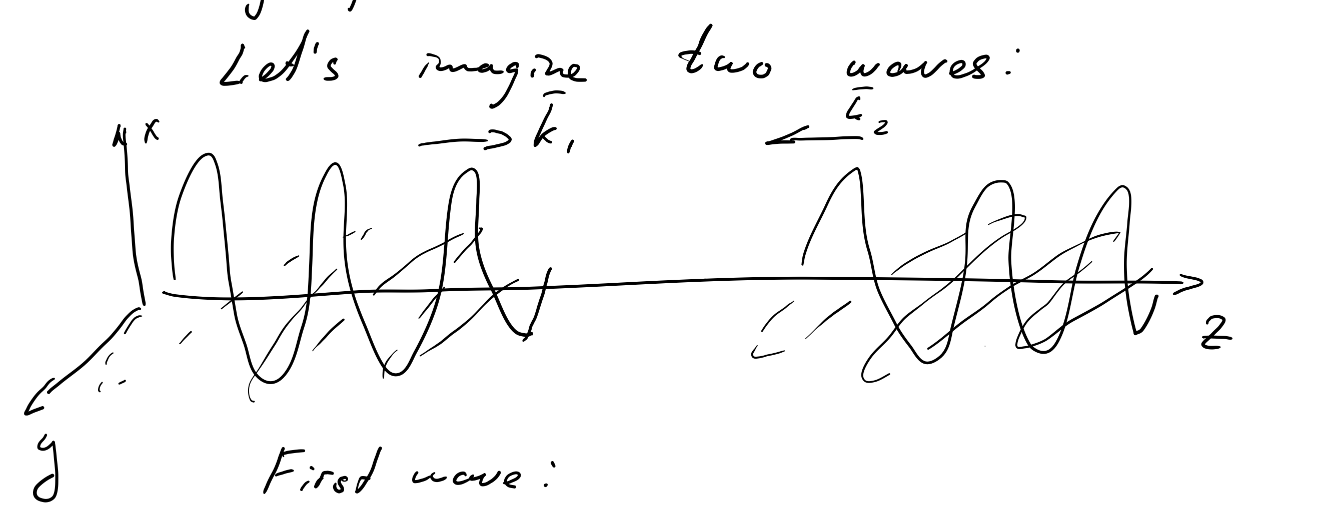
Standing waves

Very special case is actually standing waves. Such waves are possible when two counter propagating waves of same frequency, amplitude and polarization are adding up.

Let's imagine two waves:



First wave:

$$\begin{cases} \bar{E}_1 (E_0 e^{-i(\omega t - kz)}, 0, 0) \\ \bar{B}_1 (0, B_0 e^{-i(\omega t - kz)}, 0) \end{cases}$$

Second wave:

$$\begin{cases} \bar{E}_2 (E_0 e^{-i(\omega t + kz)}, 0, 0) \\ \bar{B}_2 (0, -B_0 e^{-i(\omega t + kz)}, 0) \end{cases}$$

$$[\bar{k}\bar{B}] = \frac{\omega}{c^2}\bar{E}$$

Ask to explain

Based on superposition principle:

$$\begin{cases} \bar{E} = \bar{E}_1 + \bar{E}_2 \\ \bar{B} = \bar{B}_1 + \bar{B}_2 \end{cases}$$

$$E_x = E_0 e^{-i(\omega t - kz)} + E_0 e^{-i(\omega t + kz)} =$$
$$= 2 E_0 \underbrace{e^{-i\omega t}}_{1} \underbrace{\left(\frac{e^{-ikz} + e^{ikz}}{2} \right)}_{2}$$

1) We will take only Re part.

2) $\frac{e^{-ikz} + e^{ikz}}{2} = \cos(kz)$

That means:

$$E_x = 2 E_0 \cos kz \cos \omega t$$

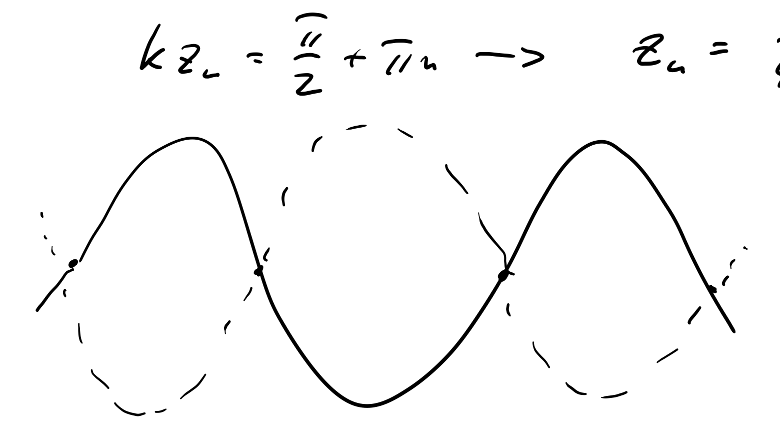
→ Amplitude is z dependent

Maxima will be at:

$$kz_n = \pi n \rightarrow \frac{2\pi}{\lambda} z_n = \pi n$$
$$z_n = n\frac{\lambda}{2}$$

Minima will be at:

$$kz_n = \frac{\pi}{2} + \pi n \rightarrow z_n = \frac{\lambda}{4} + n\frac{\lambda}{2}$$



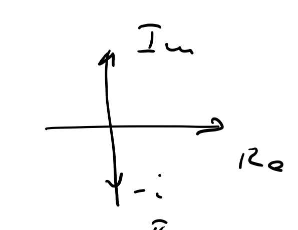
It is clear that $\lambda_{standing} = \frac{\lambda_{running}}{2}$

Let's see what happens with magnetic component:

$$B_y = B_0 e^{i(\omega t - kz)} - B_0 e^{i(\omega t + kz)} =$$
$$= B_0 e^{i\omega t} \frac{e^{ikz} - e^{-ikz}}{-2i} \cdot 2i$$

$$B_y = 2(-i) B_0 e^{i\omega t} \sin kz$$

$$B_y = 2 B_0 e^{i(\omega t - \frac{\pi}{2})} \sin kz \ ?$$



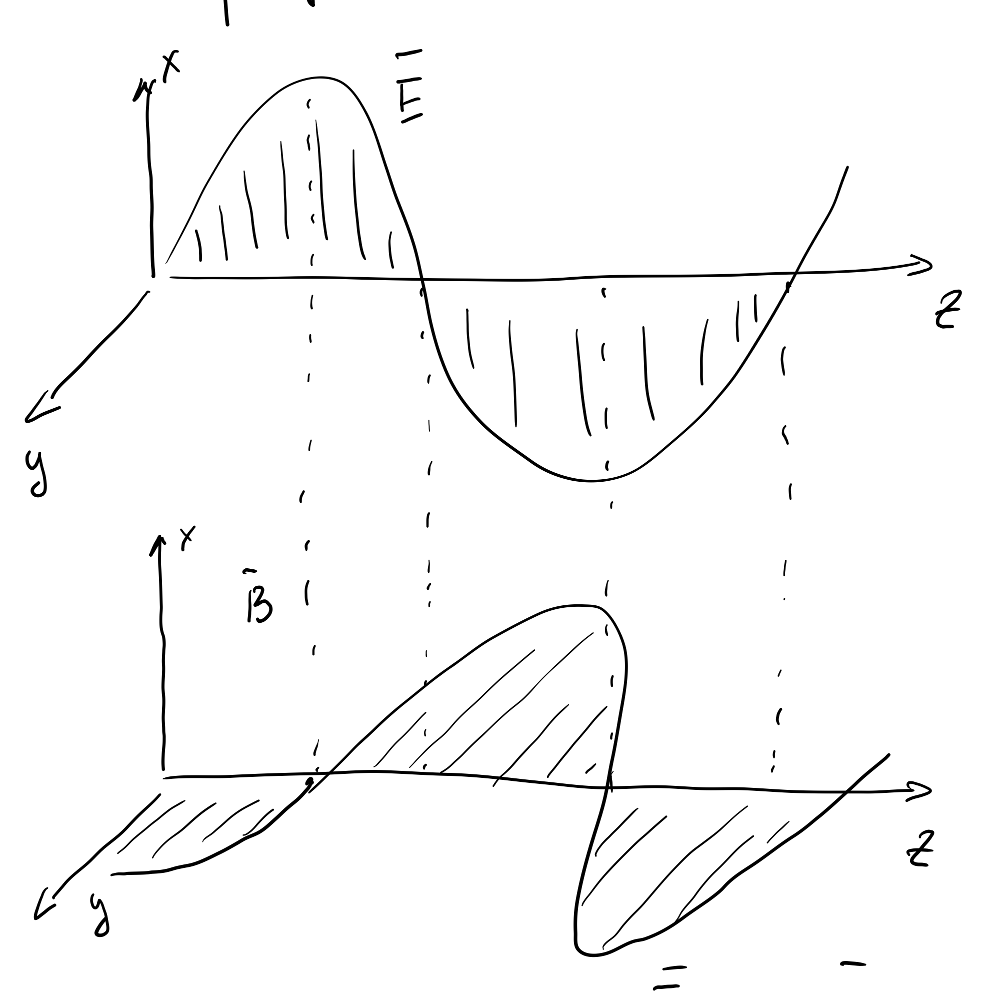
$$-i = e^{-i\frac{\pi}{2}}$$

$$B_y = 2B_0 \cos(\omega t - \tfrac{\pi}{2}) \sin kz$$

$$B_y = 2B_0 \sin \omega t \sin kz$$

What does it mean?

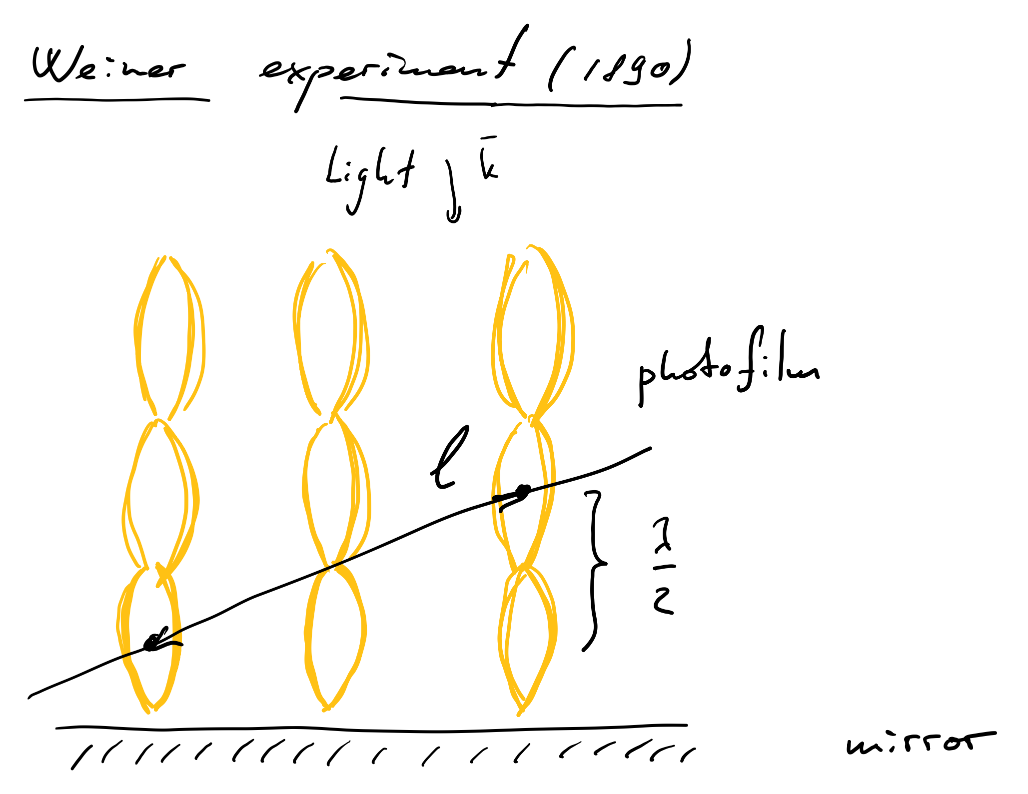
$$\begin{cases} E_x = 2E_0 \cos kz \cos \omega t \\ B_y = 2B_0 \sin kz \sin \omega t \end{cases}$$



$\bar{E}$ and $\bar{B}$ are shifted by $\frac{\pi}{2}$ or $\frac{\lambda}{4}$

Why? What about energy conservation law?

<u>Weiner experiment (1890)</u>



If $\ell = 1$ mm $\quad \alpha = 1'$

$\alpha \ell = \frac{\lambda}{2}$ → Why?

$$\lambda = 2\alpha\ell = 2 \cdot \frac{10^{-3}}{60} \cdot \frac{1}{60} m \sim \frac{1}{2} \cdot 10^{-6} = 500 nm$$

($\frac{1}{60}$ — minutes to degrees; $\frac{1}{60}$ — degrees to radians)

<u>Demonstration</u>

Lecher line $\quad$ Ernst Lecher (1909)